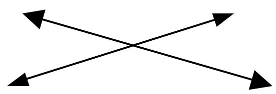
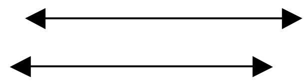
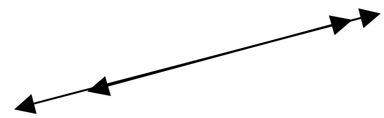
Solving a System by Substitution

Start the lesson by doing warm-up. The last problem on the warm-up (other) is the opening to this lesson.

Warm-up Question "Other"
How many distinct ways can the two lines lay in the same plane? [3]

		
The two lines can meet at one point	The two lines can be parallel	The two lines can be one on the other.

Definition: **System of equations**: a set of two of more equations.

Definition: **Solution to a system of two linear equations**: The solution to this type of system is the point of intersection, however it could also have no solution (can you name when that happens?). And it can also have infinitely many solutions (can you name when that happens?).

Example one is given because it is a problem that students are familiar with and should be able to do easily.

Ex. 1 Find the *x*- and *y*-intercept of the line $2x + 3y = 6$.

Solve by Substitution:

$$y = 0$$
$$2x + 3y = 6$$
$$2x + 3(0) = 6$$
$$2x = 6$$
$$x = 3$$

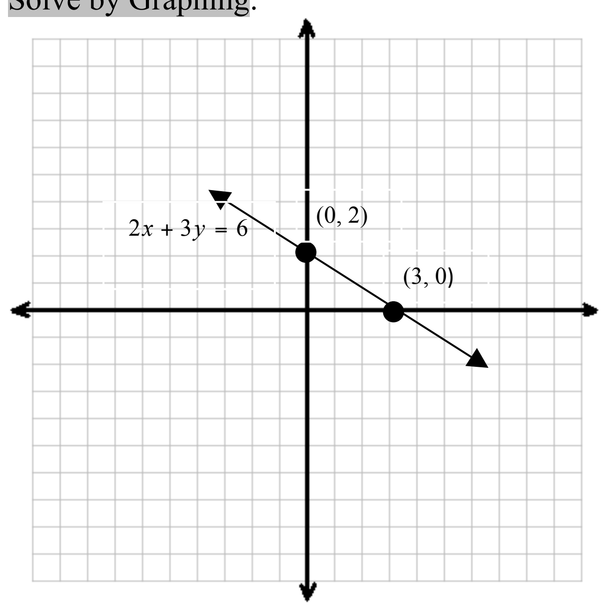
The *x*-intercept is (3, 0)

Solve by Substitution:

$$x = 0$$
$$2x + 3y = 6$$
$$2(0) + 3y = 6$$
$$3y = 6$$
$$y = 2$$

The *y*-intercept is (0, 2)

Solve by Graphing:



"$y = 0$ and $x = 0$ can be considered a value (like in the example) or a line. Knowing this can you name the three linear equations in this example?"

$$\begin{bmatrix} 2x + 3y = 6 \\ y = 0 \\ x = 0 \end{bmatrix}$$

Solving a System by Substitution

We can take these three equations and think of them as a system with a solution. For example:

"What is the solution to the system $\begin{cases} 2x+3y=6 \\ y=0 \end{cases}$?" [(3, 0)] **refer to the graph in example 1*

"What is the solution to the system $\begin{cases} 2x+3y=6 \\ x=0 \end{cases}$?" [(0, 2)] ** refer to the graph in example 1*

"What is the solution to the system $\begin{cases} x=0 \\ y=0 \end{cases}$?" [(0, 0)] **refer to the graph in example 1*

Ex. 2 "You Try" Find the *x*- and *y*-intercept of the line $3x+4y=12$.

"Name the three linear equations that we have for this example?" $\begin{bmatrix} x=0 \\ y=0 \\ 3x+4y=12 \end{bmatrix}$

"Solve the three following systems of equations.

a) $\begin{cases} 3x+4y=12 \\ y=0 \end{cases}$ b) $\begin{cases} 3x+4y=12 \\ x=0 \end{cases}$ c) $\begin{cases} y=0 \\ x=0 \end{cases}$

a) Solve by Substitution:

$$y=0$$
$$3x+4y=12$$
$$3x+4(0)=12$$
$$3x+0=12$$
$$3x=12$$
$$x=4$$

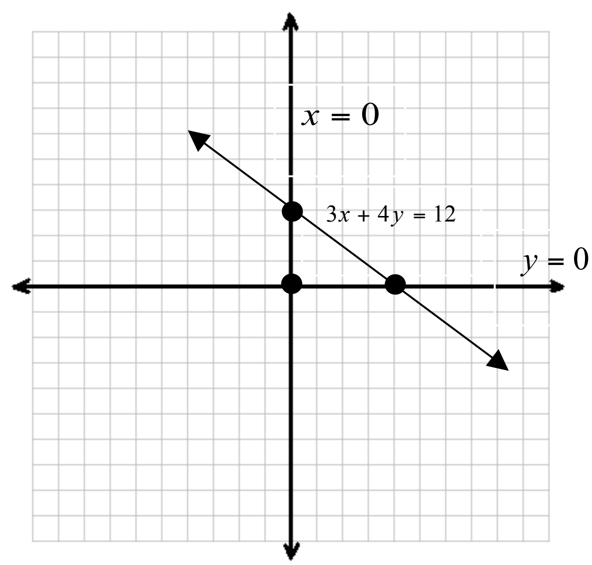
The solution to this system is (4, 0)

b) Solve by Substitution:

$$x=0$$
$$3x+4y=12$$
$$3(0)+4y=12$$
$$0+4y=12$$
$$4y=12$$
$$y=3$$

The solution to this system is (0, 3)

a, b, and c) Solve by Graphing:



The solution to the system $\begin{cases} y=0 \\ x=0 \end{cases}$ is (0, 0)

Solving a System by Substitution

Ex. 3 Given this system, solve using substitution and by graphing.

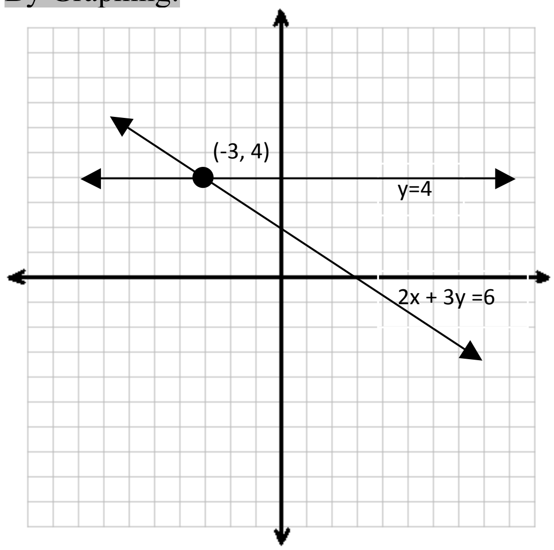
$$y = 4$$
$$2x + 3y = 6$$

By Substitution:

$$y = 4$$
$$2x + 3y = 6$$
$$2x + 3(4) = 6$$
$$2x + 12 = 6$$
$$2x + 12 - 12 = 6 - 12$$
$$2x = -6$$
$$x = -3$$

Solution point: $(-3, 4)$

By Graphing:



The point of intersection is (-3, 4)

Ex. 4 "You Try" Given this system solve for the solution point by substitution and by graphing.

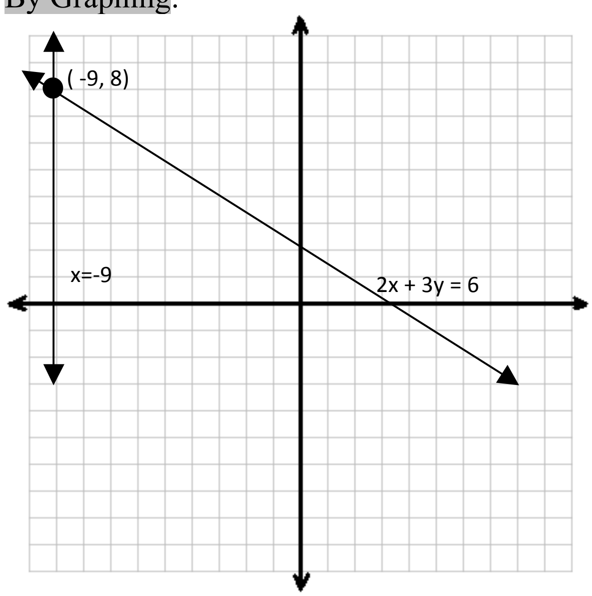
$$x = -9$$
$$2x + 3y = 6$$

By Substitution:

$$x = -9$$
$$2x + 3y = 6$$
$$2(-9) + 3y = 6$$
$$-18 + 3y = 6$$
$$-6 + 18 + 3y = 6 + 18$$
$$3y = 24$$
$$y = 8$$

Solution point: $(-9, 8)$

By Graphing:



The point of intersection is $(-9, 8)$

Solving a System by Substitution

Ex. 5 Solve the system of equations using both substitution and by graphing:

$$y = 2x$$
$$x + 2y = 10$$

"What is the difference between this example an the other examples?" [both equations have an x and a y]

"Do we know how to solve one equation with two variable?" [no]

"We need to have an equation with just one variable. Since $y = 2x$ we can **substitute** $2x$ for y in the second equation. Once we have the value of one variable, we can find the value of the second variable."

By Substitution:

Solve for *x*.

① $y = 2x$

② $x + 2y = 10$

$$x + 2(2x) = 10$$
$$x + 4x = 10$$
$$5x = 10$$
$$x = 2$$

"Remember, we need a point, an *x* and a *y* value. What should we do to find the value of *y*? [substitute 2 for *x*]

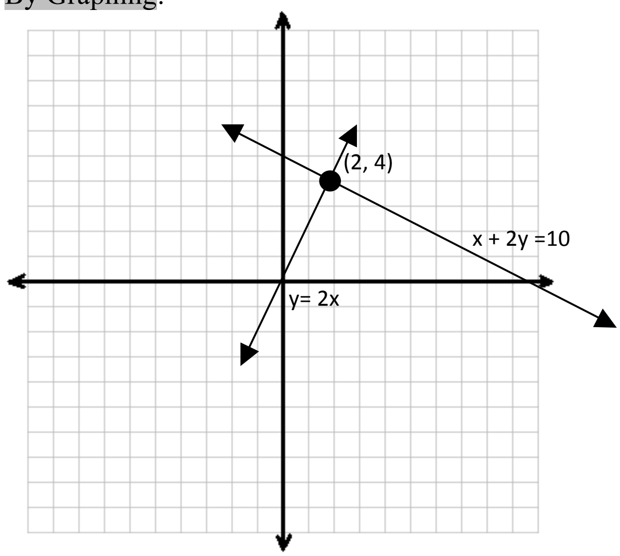
$$x = 2$$

① $y = 2x$

$$y = 2(2)$$
$$y = 4$$

Solution point: $(2, 4)$

By Graphing:



The point of intersection is (2, 4), therefore the solution is (2, 4)

Solving a System by Substitution

Ex. 6 "You Try" Given this system solve for the solution point by substitution and by graphing.

$$y = \frac{1}{2}x$$
$$x + 2y = 10$$

Using Substitution:

① $y = \frac{1}{2}x$

② $x + 2y = 10$

$$x + 2\left(\frac{1}{2}x\right) = 10$$
$$x + x = 10$$
$$2x = 10$$
$$x = 5$$

Now, solve for y

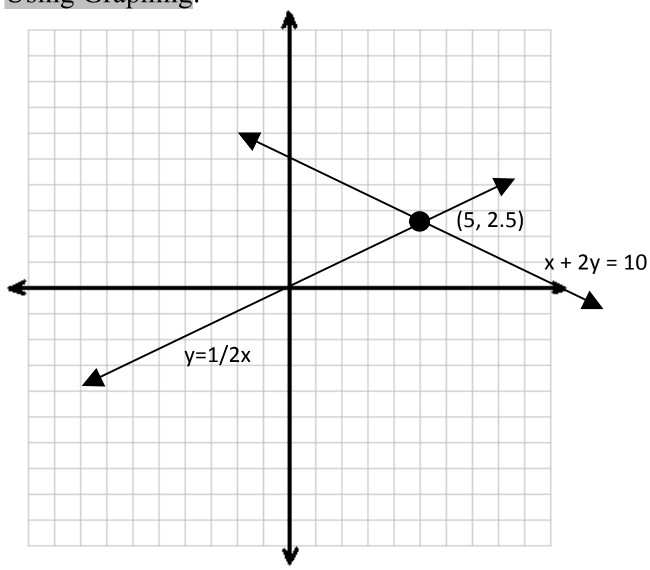
$x = 5$

② $y = \frac{1}{2}x$

$$y = \frac{1}{2}(5)$$
$$y = 2\frac{1}{2}$$

Solution point $(5, 2\frac{1}{2})$

Using Graphing:



The point of intersection is $(5, 2\frac{1}{2})$

Solving a System by Substitution

Ex. 7 Solve the system of equations by substitution and by graphing.

$$y = \frac{1}{2}x + 3$$
$$4x + 6y = 4$$

By Substitution:

① $y = \frac{1}{2}x + 3$

② $4x + 6y = 4$

$4x + 6(\frac{1}{2}x + 3) = 4$

$4x + 6(\frac{1}{2}x) + 6(3) = 4$

$4x + 3x + 18 = 4$

$7x + 18 = 4$

$7x + 18 - 18 = 4 - 18$

$7x = -14$

$x = -2$

Now, solve for y

$x = -2$

② $y = \frac{1}{2}x + 3$

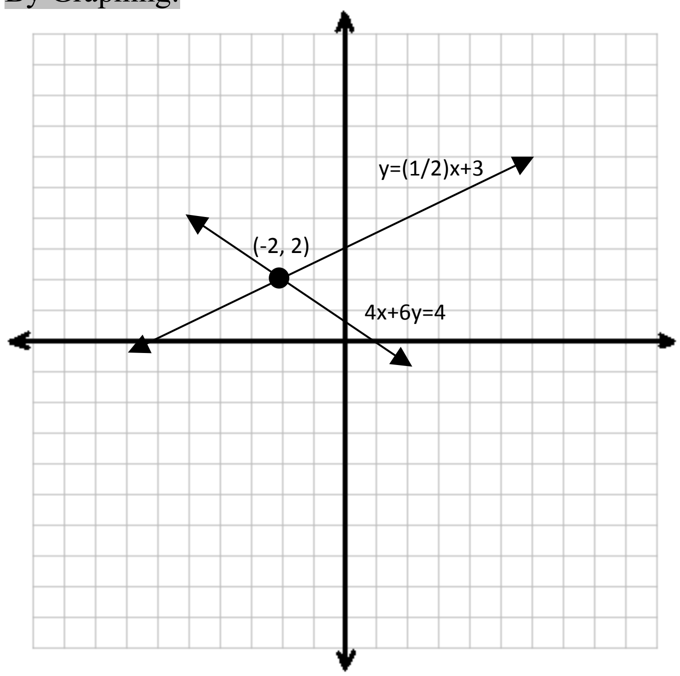
$y = \frac{1}{2}(-2) + 3$

$y = -1 + 3$

$y = 2$

Solution point: $(-2, 2)$

By Graphing:



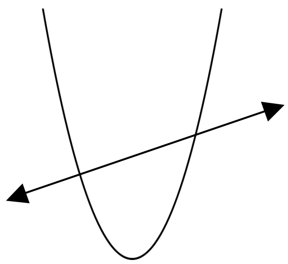
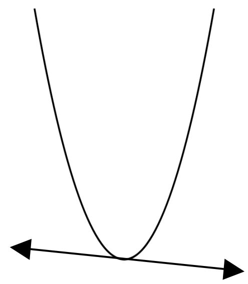
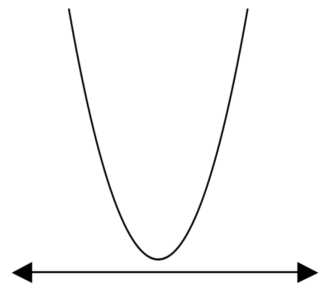
The point of intersection is $(-2, 2)$.

Solving a System by Substitution

At this point of the lesson, you will hand out a transparency of a parabola. This will be a **"think, pair, share" (TPS)** activity. Pose the following question to your students.

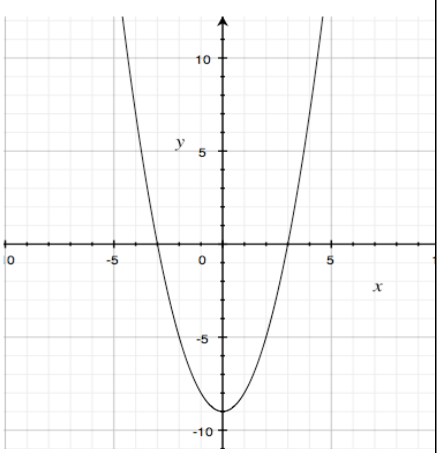
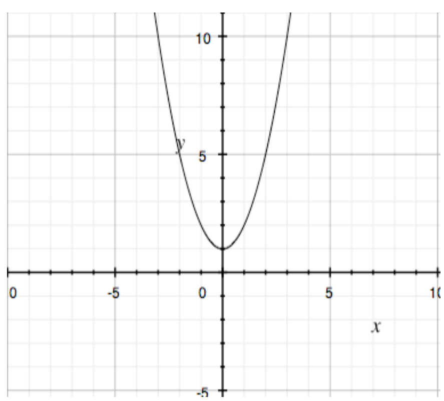
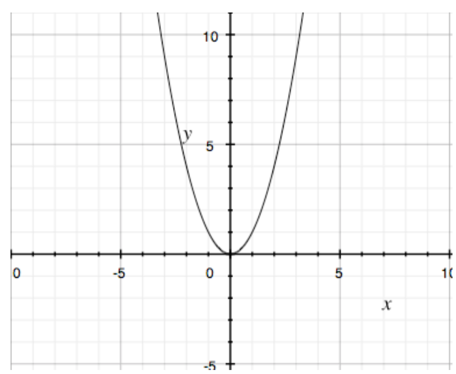
"How many ways can the parabola and one line lay in a plane?" [3]

Have students come forward and display their answers.

		
Two intersection points, 2 solutions	One intersection point, 1 solution	No intersection points, no solutions

In their TPS groups have students come up with the solutions to the following example.

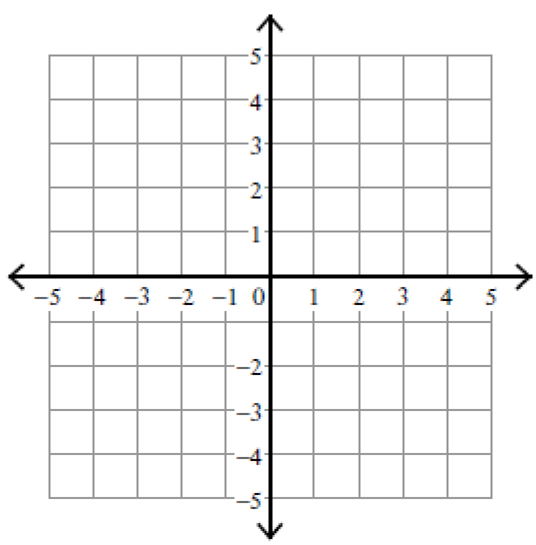
Ex. 8 Can you name the solutions points of the given system of equations by looking at the graph?

a) $\begin{cases} y = x^2 - 9 \\ y = 0 \end{cases}$	b) $\begin{cases} y = x^2 + 1 \\ y = 0 \end{cases}$	c) $\begin{cases} y = x^2 \\ y = 0 \end{cases}$
 Solution: (-3, 0) and (3, 0)	 No solution	 Solution: (0, 0)

Solving a System by Substitution

Warm-Up

CST/CAHSEE:	Review:
What is the y-intercept of the graph $4x+2y=12$? A. -4 B. -2 C. 6 D. 12	Determine which of the following points lie on the line $x+2y=5$. A. $(1,1)$ B. $(-1,-1)$ C. $(1,2)$ D. $(2,1)$

Current:	Other:
Graph the two lines on the same plane. $y=3x-4$ $y=-3x+2$ 	Using the two lines on the transparencies, find the number of original ways you can place them in a plane.

Today's Objective/Standard: Algebra 1 9.0